KINEMATIC INTERPRETATION OF THE MOTION OF A BODY WITH A FIXED POINT

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The author applies the geometrical method of investigation of the motion of a body with a fixed point [2] to the earlier solution obtained in [1]. Moving and stationary hodograph of the angular velocity of the body is constructed for all values of the parameters of the system.

Motion of a rigid body with a fixed point in the homogeneous gravity field is described by

$$A\frac{dp}{dt}=(B-C)\,qr+(e_2\nu_3-e_3\nu_2)\,\Gamma,\qquad \frac{d\nu_1}{dt}=r\nu_2-q\nu_3\qquad \begin{pmatrix}1&2&3\\A&B&C\\p&q&r\end{pmatrix}\tag{0.1}$$

where e_1, e_2, e_3 denote the unit vector with its origin at the fixed point and directed towards the center of mass; Γ is the mass of the body multiplied by the distance between the center of mass and the fixed point and ν_1, ν_2, ν_3 denote the unit gravity vector. Other notation follows the accepted usage.

When the conditions

$$e_2=0,\quad e_1=\cos\delta,\quad e_3=\sin\delta,\quad \operatorname{tg}\delta=-\left(\frac{A\,(C-B)\,(2C-A)^3}{C\,(A-B)\,(C-2A)^3}\right)^{1/2}\quad (C>2A>2B)\tag{0.2}$$

hold (it was shown in [1] that they hold e.g. in the case of a rigid body with cavities filled with fluid), Eqs. (0.1) have the following solution [1]:

$$Ap=\beta\,(\cos\mu+\chi\sin\mu\cos\sigma),\qquad q=\beta\left(\frac{(C-2A)(2C-A)}{3ACH}\right)^{1/2}\sin\sigma$$

$$Cr=\beta\,(\sin\mu-\chi\cos\mu\cos\sigma),$$

$$\nu_1H=(3AC-2AB+BC)\cos\mu+3AC\,\chi\sin\mu\cos\sigma-3A\,(C-B)\cos\mu\cos^2\sigma\tag{0.3}$$

$$\nu_2H^{1/2}=\sqrt{3AC\,(C-2A)\,(2C-A)}\left[B-3\chi\left(\frac{AC\,(A-B)\,(C-B)}{(C-2A)\,(2C-A)}\right)^{1/2}\cos\sigma\right]\sin\sigma$$

$$\nu_3H=(3AC-2BC+AB)\sin\mu-3AC\,\chi\cos\mu\cos\sigma-3C\,(A-B)\sin\mu\cos^2\sigma$$

$$\cos\mu=\left(\frac{C\,(A-B)\,(C-2A)}{H\,(C-A)}\right)^{1/2},\qquad \sin\mu=\left(\frac{A\,(C-B)\,(2C-A)}{H\,(C-A)}\right)^{1/2}$$

$$\chi=\left(\frac{3AC-2B\,(A+C)}{3AC}\right)^{1/2}$$

$$\beta^2=\frac{9A^2C^2\Gamma\sqrt{C-A}}{\sqrt{H\,[A\,(C-B)\,(2C-A)^3+C\,(A-B)\,(C-2A)^3]}},\qquad H=3AC-B\,(A+C)$$

while

$$\frac{d\sigma}{d\tau} = u_* + \cos\sigma \tag{0.4}$$

$$\tau = t\beta\,\frac{A+C}{3AC}\left(\frac{3(A-B)(C-B)}{H}\right)^{1/2} t,$$

$$u_* = \left(\frac{(C-2A)(2C-A)(3AC-2AB-2BC)}{3(A+C)^2(A-B)(C-B)}\right)^{1/2}$$

gives the time relationship.

Let us discuss this solution from the kinematic point of view.

1. Let us introduce the following dimensionless parameters

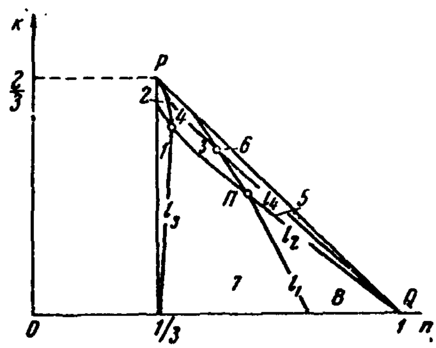

Fig. 1

$$k = \frac{2B}{A+C}, \qquad n = \frac{C-A}{C+A}$$

$$(k > 0,\ 1 > n > 0)$$

Then

$$A = \frac{1-n}{k}B, \qquad C = \frac{1+n}{k}B$$

Condition (0.2) yields $1+n > 2(1-n) > 2k$, from which

$$1/3 < n < 1, \qquad 0 < k < 2/3, \qquad n + k < 1$$

follows.

Fig. 1 shows the (n, k)-parametric space satisfying the above inequalities. Henceforth we shall call this space, in short, the "triangle".

In the n, k notation, solution (0.3) become

$$p = \frac{\beta}{B}\frac{k}{1-n}(\cos\mu + \chi\sin\mu\cos\sigma), \qquad r = \frac{\beta}{B}\frac{k}{1+n}(\sin\mu - \chi\cos\mu\cos\sigma)$$

$$q = \frac{\beta}{B}k\left(\frac{9n^2-1}{3(1-n^2)h}\right)^{1/2}\sin\sigma \tag{1.1}$$

$$\nu_1 h = [3 - 3n^2 + k(3n-1)]\cos\mu + 3(1-n^2)\chi\sin\mu\cos\sigma -$$
$$- 3(1-n)(1+n-k)\cos\mu\cos^2\sigma$$

$$\nu_2 h\sqrt{h} = [k\sqrt{3(1-n^2)(9n^2-1)} - 3\sqrt{(1-n^2)(3-3n^2-4k)(1-2k+k^2-n^2)}\ \times$$
$$\times \cos\sigma]\sin\sigma \tag{1.2}$$

$$\nu_3 h = [3 - 3n^2 - k(3n+1)]\sin\mu - 3(1-n^2)\chi\cos\mu\cos\sigma - 3(1+n)(1-n-k)\sin\mu\cos^2\sigma$$

$$\cos\mu = \left(\frac{(1+n)(3n-1)(1-n-k)}{2nh}\right)^{1/2}, \quad \sin\mu = \left(\frac{(1-n)(3n+1)(1+n-k)}{2nh}\right)^{1/2}$$

$$\chi = \left(1 - \frac{4k}{3(1-n^2)}\right)^{1/2} \tag{1.3}$$

$$\beta^3 = \frac{9\Gamma B(1-n^2)^2\sqrt{2n}}{k\sqrt{h}[(1-n)(1+n-k)(3n+1)^3 + (1+n)(1-n-k)(3n-1)^3]}$$

$$\tau = \frac{\nu}{B}\frac{2k}{3-3n^2}\sqrt{3h^{-1}(1-2k+k^2-n^2)}\,t, \quad h = 3 - 3n^2 - 2k$$

$$u_* = \left(\frac{(9n^2-1)(3-3n^2-4k)}{12(1-2k+k^2-n^2)}\right)^{1/2} \tag{1.4}$$

We note that for n, k lying on the curve

$$l_1(n, k) \equiv 9n^2 + 6k - 5 = 0$$

$u_* = 1$. This parabola touches the hypotenuse at the point P and divides the triangle into two regions; on the left-hand side of the parabola we have $u_* < 1$, and on the right side we have $u_* > 1$.

2. Let us now introduce the following relation:

$$(p, q, r) = \frac{2}{3}\frac{\nu}{B}\frac{k}{1-n^2}(p', q', r')$$

Omitting the primes we obtain, in place of (1.1),

$$p = {}^3/_2\,(1+n)(\cos\mu + \chi\sin\mu\cos\sigma), \qquad r = {}^3/_2\,(1-n)(\sin\mu - \chi\cos\mu\cos\sigma) \tag{2.1}$$

$$q = {}^1/_2\sqrt{3h^{-1}(9n^2-1)(1-n^2)}\,\sin\sigma \tag{2.2}$$

Eliminating σ from (2.1) we obtain

$$\frac{2}{3(1+n)}p\cos\mu + \frac{2}{3(1-n)}r\sin\mu = 1 \tag{2.3}$$

Expression (2.1) yields also another expression, viz.

$$\frac{2}{3(1+n)}p\sin\mu - \frac{2}{3(1-n)}r\cos\mu = \chi\cos\sigma \tag{2.4}$$

Eliminating now σ from (2.4) and (2.2) we obtain

$$\left(\frac{2\sin\mu}{3(1+n)\chi}p - \frac{2\cos\mu}{3(1-n)\chi}r\right)^2 + \frac{4h}{3(9n^2-1)(1-n^2)}q^2 = 1 \tag{2.5}$$

In the (p, q, r) – space associated with the body, Eq. (2.3) defines the plane parallel to q and (2.5) defines an elliptic cylinder. The line of intersection of these two surfaces constitutes the moving hodograph.

Rotation of the p, q, r-axes about q by the angle ε

$$\operatorname{tg}\varepsilon = \frac{1-n}{1+n}\operatorname{tg}\mu$$

represents the transformation to the s, q, s'-axes and (2.5) becomes, in this system

$$\frac{s'^2}{l^2\chi^2} + \frac{q^2}{l_1^2} = 1 \tag{2.6}$$

$$l = \frac{3(1-n^2)}{2\sqrt{(1-n)^2\sin^2\mu + (1+n)^2\cos^2\mu}}$$

$$l_1 = \frac{1}{2}\sqrt{3h^{-1}(9n^2-1)(1-n^2)}$$

This is an elliptic cylinder whose axis is s.

Fig. 2 shows a moving axoid-cone, and the ellipse produced by the intersection of the cylinder (2.6) with the plane (2.3) is its directrix.

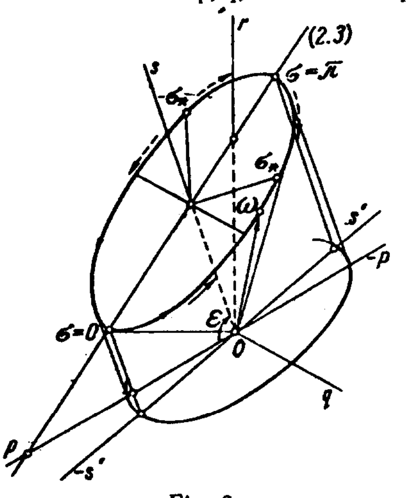

Fig. 2

We note that the points of the hodograph lying on the straight line (2.3) satisfy the condition $q = 0$ and this, according to (2.2), takes place when $\sigma = 0, \pi$. From (1.3) we find that $\sin^2\mu - \cos^2\mu > 0$ and that $\frac{1}{4}\pi < \mu < \frac{1}{2}\pi$.

Formulas (2.1) yield now $p > 0$, $r > 0$, for $\sigma = 0$ and $p < 0$, $r > 0$ for $\sigma = \pi$ and these points, which belong to the moving hodograph, are indicated on Fig. 2.

From (0.4) we have

$$\tau = \int_{\sigma_0}^{\sigma} \frac{d\sigma}{u_* + \cos\sigma}$$

When $u_* > 1$ and $d\sigma/d\tau > 0$, σ increases without bounds, and the whole ellipse serves as the hodograph, We can therefore assume that at the initial instant $\sigma = 0$ and, that it in-increases so that q goes from zero to the positive values.

When $u_* = 1$, the extremity of the vector of angular velocity approaches the point $\sigma = \pi$ asymptotically. Starting from an arbitrary initial position, ω will move in the direction shown on Fig. 2, since $d\sigma/d\tau > 0$.

When $0 < u_* < 1$, such values of $\pm\sigma$ can be found within the intervals $(-\pi, -\frac{1}{2}\pi)$ and $(\frac{1}{2}\pi, \pi)$, that $\cos(\pm\sigma_*) = -u_*$. The corresponding points on the ellipse are asymptotic for the extremity of ω. If at the initial instant $u_* + \cos\sigma > 0$, we have

$$\cos\sigma > -u_* = \cos\sigma_*, \qquad -\sigma_* < \sigma < \sigma_*, \qquad \frac{d\sigma}{d\tau} > 0$$

while if $u_* + \cos\sigma < 0$, we have

$$\cos\sigma < -u_* = \cos\sigma_*, \qquad -\pi < \sigma < -\sigma_*, \quad \sigma_* < \sigma < \pi, \qquad \frac{d\sigma}{d\tau} < 0$$

Broken arrows on Fig. 2 show the direction of motion of ω. Thus the character of the motion depends on the position of the point (n, k) corresponding to the given values of A, B and C within the triangle.

3. It was shown in [2] that the knowledge of the following three magnitudes is required for construction of the stationary hodograph: $\omega_\zeta(\sigma) = \omega(\sigma)\,\nu(\sigma)$ and $\omega_\rho^2(\sigma) = \omega^2(\sigma) - \omega_\zeta^2(\sigma)$ which represent, respectively, the axial and radial component of the angular velocity, and the third cylindrical coordinate α defined by

$$\omega_\rho^2 \frac{d\alpha}{d\sigma} = \begin{vmatrix} \nu_1 & \nu_2 & \nu_3 \\ p & q & r \\ dp/d\sigma & dq/d\sigma & dr/d\sigma \end{vmatrix}$$

Putting now

$$\cos\sigma = u \tag{3.1}$$

we obtain, taking (1.1) to (1.3) into account, the required magnitudes

$$\omega^2 = \frac{12}{h}(1 - n^2 + k^2 - 2k)\left[u^2 + \frac{3}{2}u_* u + \frac{(1-n^2)(1+9n^2) - 12kn^2}{8(1-n^2+k^2-2k)}\right] \tag{3.2}$$

$$\omega_\zeta = \frac{3}{h^2}(1 - n^2 + k^2 - 2k)\left[3(1-n^2)u^2 + (h+4k)u_* u + \right.$$

$$\left. + \frac{3(1-n^2)^2(2-3k) + 4(1-3n^2)k^2}{2(1-n^2+k^2-2k)}\right] \tag{3.3}$$

$$\omega_\rho^2 = \omega^2 - \omega_\zeta^2 = N(u + u_*)^2(u + u_1)(u_2 - u) \tag{3.4}$$

$$\frac{d\alpha}{d\sigma} = L\frac{(u+u_3)(u_4-u)}{(u+u_*)(u+u_1)(u_2-u)} \tag{3.5}$$

where

$$u_{1,2} = \frac{2}{3}\frac{u_*}{1-n^2}\left[\frac{2h}{\sqrt{2(9n^2-1)}} \pm k\right], \qquad u_{3,4} = \frac{u_*}{4}\left\{\left(1 + 16\frac{4-3k}{9n^2-1}\right)^{1/2} \pm 1\right\} > 0$$

$$N = \frac{81}{h^4}(1-n^2)^2(1-n^2+k^2-2k)^2, \qquad L = \frac{h^{3/2}}{2\sqrt{3(1-n^2+k^2-2k)}} \tag{3.6}$$

The quantity u_* is given by (1.4) and $u_{1,2}$ are positive within the triangle.

Eliminating u from (3.2) and (3.3) and writing the left-hand side of (3.2) as $\omega_\rho{}^2 + \omega_\zeta{}^2$, we obtain

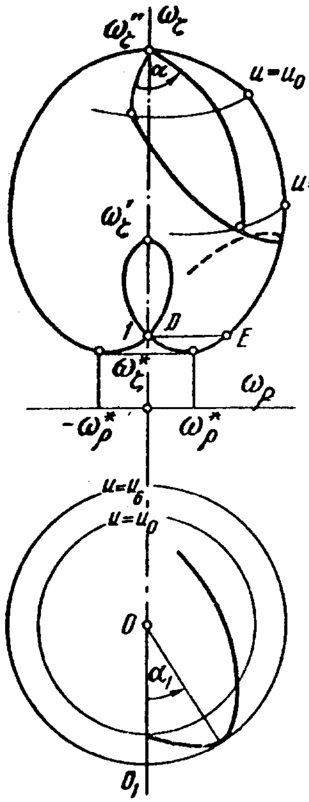

Fig. 3

$$[\omega_\rho{}^2 + (\omega_\zeta - c_1)^2 - c_2]^2 - c_3(\omega_\zeta + c_4) = 0$$

$$c_1 = \frac{2}{3}\,\frac{h}{1-n^2}, \qquad c_2 = \frac{(1+3n^2)(h-2k)^2}{12(1-n^2)^2}$$

$$c_3 = \frac{(9n^2-1)(h-2k)^3}{27(1-n^2)^3} \tag{3.7}$$

$$c_4 = \frac{3(5+3n^2)(h-2k)-32h}{48(1-n^2)}$$

This curve lies in the $(\omega_\rho\omega_\zeta)$-plane and represents the meridian of the surface of revolution containing the stationary hodograph. Let us construct this hodograph. When $\omega_\rho = 0$, (3.7) yields the following polynomial

$$f(\omega_\zeta) \equiv (\omega_\zeta - 1)^2(\omega_\zeta - \omega_\zeta')(\omega_\zeta - \omega_\zeta'')$$

where

$$\omega_\zeta',\ \omega_\zeta'' = \frac{3h-2k}{3(1-n^2)} \mp \frac{h-2k}{3(1-n^2)}\sqrt{\frac{9n^2-1}{2}} > 1 \tag{3.8}$$

The derivative $d\omega_\zeta/d\omega_\rho$ obtained from (3.7) becomes zero at $\omega_\rho = 0$ and $\omega_\rho = \pm\omega_\rho{}^*$ and

$$\omega_\rho{}^* = \frac{h-2k}{16(1-n^2)}\sqrt{-9n^2+34n^2-{}^{11}/_3} \tag{3.9}$$

Considering the second derivative we find, that the curve has a maximum at the points $(0, \omega_\zeta{}')$ and $(0, \omega_\zeta{}'')$, and a minimum at the points $(\pm\omega_\rho{}^*, \omega_\zeta{}^*)$. The value of $\omega_\zeta{}^*$ is obtained from (3.7) putting $\omega_\rho = \omega_\rho^*$ (3.10)

$$\omega_\zeta{}^* = (1-n^2)^{-1}\,[{}^2/_3\,h - {}^1/_{16}\,(5+3n^2)\,(h-2k)] < 1$$

Fig. 3 shows the curve (3.7). A question arises whether the hodograph will pass through the point D (Fig. 3). Putting $\omega_\zeta = 1$ in (3.3), we obtain the following two values

$$u_{(1)} = -\frac{2k}{3(1-n^2)}\,u_*, \qquad u_{(2)} = -u_*$$

From (3.4) it follows that $\omega_\rho(u_{(1)}) \neq 0$ and $\omega_\rho(u_{(2)}) = 0$, consequently the values obtained correspond to the points E and D. The stationary hodograph will pass through the point D if $|u_{(2)}| = |u_*| < 1$, and this condition is fulfilled when n, k lie to the left of the curve l_1 (Fig. 1). Arguments given in Section 2 lead to conclusion that D is achieved asymptotically as $t \to \infty$.

In addition we shall write the following Expressions

$$d\omega_\rho{}^2 / du = -4N\,(u+u_*)(u+u_5)(u-u_6) \tag{3.11}$$

$$\operatorname{tg}\varkappa = \frac{\omega_\rho\,d\alpha}{d\omega_\rho} = L\,\frac{(u+u_3)(u_4-u)}{2(u+u_5)(u-u_6)\sqrt{1-u^2}} \tag{3.12}$$

$$u_{5,6} = u_*\left[\left(\frac{(1-n^2+2k)^2}{16(1-n^2)^2} + \frac{2k^2-k(3n^2+5)+4(1-n)}{(1-n^2)(9n^2-1)}\right)^{1/2} \pm \frac{1-n^2+2k}{4(1-n^2)}\right] > 0 \tag{3.13}$$

4. The type of motion depends, naturally, on the character of $u_1, \ldots, u_6$ defined by Formulas (3.6) and (3.13).

We find that $u_1 > 1$ and $u_3 > 1$ within the triangle, while u_2 and u_4 both become equal to unity on the hyperbola

$$l_2(n,k) \equiv 9n^2 - 18\,(k - {}^2/_3)^2 - 1 = 0$$

which touches the leg of the right angle at the point P and has the direction cosine equal to $-\,3/4$ on the approach to Q (l_2 and subsequent curves are shown on Fig. 1). With exception of the points on l_2, $u_2 > 1$ within the triangle, $u_4 > 1$ above the hyperbola and $u_4 < 1$ below it.

On the curves $l_3(n,k) \equiv u_5(n,k) - 1 = 0$ and $l_4(n,k) \equiv u_6(n,k) - 1 = 0$, we have $u_5 = u_6 = 1$. The curve l_4 touches the hypotenuse at the point Q and passes to the point P, forming the angle arc tg 2/3 with the vertical leg of the right angle. The curve l_3 passes through P at the same angle but is situated near the vertical leg and touches it on the approach to the abscissa; $u_6 > 1$ above l_4 and $u_6 < 1$ below it, $u_5 > 1$ to the right of l_3 and $u_5 < 1$ on the other side of l_3.

All these curves divide the triangle into 8 regions. Both, the moving and the stationary hodograph or, in other words, the whole pattern of motion of the body, will depend on the particular region in which n and k, known for each specific example, are found.

5. The motion can now be interpreted in the following order. We construct the moving hodograph using the Formulas of Section 2. Expressions (2.1) and (2.2) make it possible to set up a correspondence between σ and the points on the moving hodograph. If the point u_* lies within the interval $(-1,1)$, then the point on the hodograph will correspond to the value $\sigma_* = \arccos(-u_*)$ and ω will approach this point asymptotically.

Using (3.8) to (3.10) we can construct the surface (3.7) for the specified values of n and k.

Let the initial value u_0 be given; $\omega_\zeta(u_0)$ can be found from (3.3). This will define the position of the moving axoid on the stationary axoid at the initial instant (Fig. 3a shows the initial parallel on the surface of revolution), while Fig. 3b represents the projection of this surface on the plane $\zeta = 0$. First (3.11) and second derivative of ω_ρ^2 show that ω_ρ^2 is maximum at $u = u_6$ and the outer circumference on Fig. 3b corresponds to the parallel $u = u_6$. The parallel corresponding to $u = u_0$ is also shown.

The angle α is counted from the radius OO_1 (Fig. 3b).

The formulas available throw some light on the character of the stationary hodograph.

From (3.12) it follows that tg $\varkappa$ becomes infinite at $u = u_6$, and at $u = \pm 1$ it approaches these parallels tangentiall. When $u = u_4$, $\varkappa = 0$ and the curve touches the radius on this parallel (we discuss here a general case; a case such as e.g. $u = u_4 = u_6$ should be considered particularly).

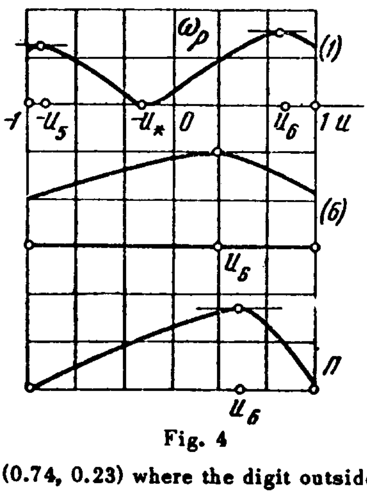

Fig. 4

The sign of the derivative $d\alpha/d\sigma|_{u=u_0}$ obtained from (3.5) will decide the left or right direction in which the curve will move away from the vertical radius.

Fig. 3 shows also a part of the curve corresponding to the case when $d\alpha/d\sigma|_0 > 0$ and tg $\varkappa|_0$ is different from zero and infinity. Angle α_1 can be found (with (3.1) taken into account) as follows:

$$\alpha_1 = L \int_{\arccos u_0}^{\arccos u_*} \frac{(\cos\sigma + u_3)(u_4 - \cos\sigma)\,d\sigma}{(\cos\sigma + u_*)(\cos\sigma + u_1)(u_2 - \cos\sigma)}$$

Depending on the value of u_*, the curve will either wrap itself around the center of the circle on Fig. 3b and approach the point D on Fig. 3a ($u_* < 1$), or remain within the annulus between two parallels $u = \pm 1$ ($u_* > 1$).

Let us follow the progress of the moving axoid along the stationary one in the following three, most typical cases. Let us consider the points 1 (0.34, 0.37) and 4 (0.74, 0.23) where the digit outside the bracket indicates the region of the triangle in which the point in question lies, and the numbers within the brackets are the values of n, k;

2 (0.57, 0.33) is the point of intersection of the curves l_1 and l_2. Below we give the values of the parameters required in constructing the flow

		$l\chi$	l_1	ε	ω_ζ'	ω_ζ''	ω_ζ^*	ω_ρ^*
1	$0.05p + 1.01r = 1$	1.32	0.16	78°	1.79	1.96	0.99	0.04
6	$0.08p + 2.52r = 1$	0.87	1.23	36°	1.18	2.04	0.92	0.20
II	$0.11p + 1.52r = 1$	1.00	0.86	45°	1.33	2.00	0.96	0.16

	c_1	c_2	c_3	c_4	u_*	u_1	u_2	u_3	u_4
1	1.44	0.19	0.01	0.99	0.16	1.25	1.16	1.05	0.96
6	1.31	0.19	0.11	0.92	1.92	2.44	1.12	2.29	1.34
II	1.33	0.16	0.07	0.96	1.00	1.66	1.00	1.49	1.00

	u_5	u_6	N	L	χ	$\omega_\rho(-1)$	$\omega_\rho(1)$	$\omega_\rho(u_6)$
1	0.93	0.77	0.37	1.44	0.66	0.37	0.42	0.50
6	2.27	0.32	0.03	1.24	0.56	0.31	0.36	0.64
II	1.45	0.45	0.14	1.33	0.58	0.00	0.00	0.59

Fig. 4 shows the graphs of $\omega_\rho(u)$ for the above cases.

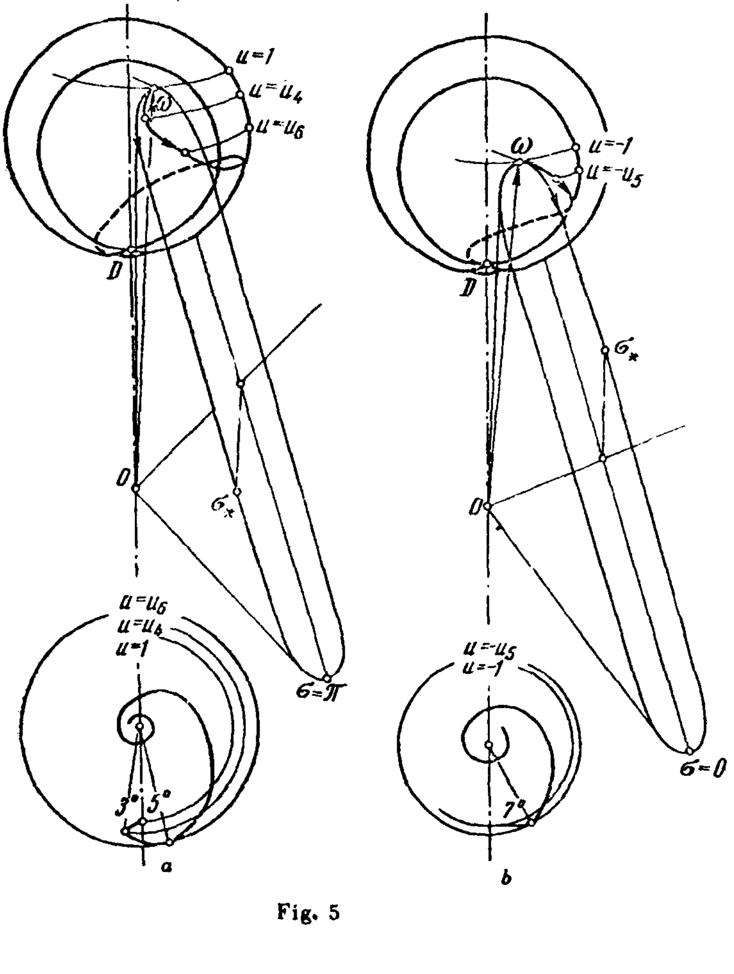

Fig. 5

P o i n t *1.* Here $u_* < 1$. Let us take $u_0 = 1$ ($\sigma_0 = 0$) as its initial value; the derivative $d\alpha/d\sigma|_{u=1} < 0$ and remains until u, decreasing in value from $u = 1$, reaches the value $u = u_4$. Then we have $d\alpha/d\sigma|\ u<u_4 > 0$. The curve (Fig. 5a) intersects the parallel $u = u_4$ at right angles ($\operatorname{tg} \varkappa|_{u=u_4} = 0$) when $\alpha \approx -3°$ and turns to the right. Maximum value of ω_ρ is reached at $\alpha \approx 5°$ (tangency with $u = u_6$). As u approaches $-u_*$, the curve begins to spiral about the center. The Figure shows both, the moving and the stationary hodograph. The point of the moving hodograph corresponding to the value $\sigma_* = 100°$ approaches the point D asymptotically as the moving axoid rolls on the surface of revolution.

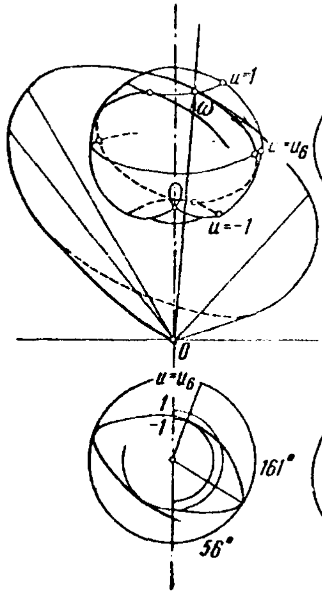

Fig. 6

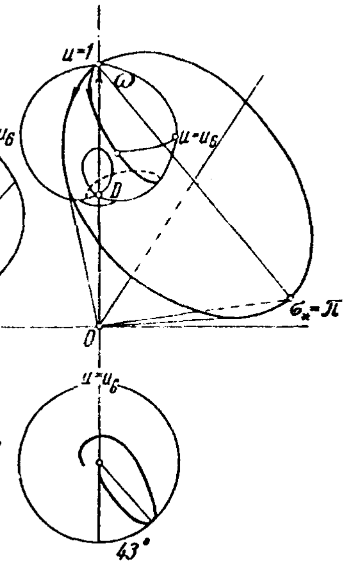

Fig. 7

Fig. 5*b* shows what happens when the value $u_0 = -1$ is taken, with everything else unchanged. The moving axoid rolls, in this case, on the inner part of the surface of revolution.

At the point 6 we have $u_* > 1$, consequently the angular velocity vector makes a complete circuit around the moving hodograph. We have taken $u_0 = 1$ as the initial value on the Fig. 6 and the stationary hodograph is contained between the parallels $u = \pm 1$. At the point *2* we have $u_* = 1$, $u_2 = 1$ and $u_4 = 1$, therefore $\omega_\rho \pm(1) = 0$. When $u_0 = 1$, the stationary hodograph originates at the apex of the surface (Fig. 7), intersects the parallel $u = u_6$ at $\alpha \approx 43°$ and approaches the point D asymptotically as $u \to -1$. At the same time the extremity of the vector ω on the moving hodograph approaches the point $\sigma_* = \pi$.

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